Baltic Balancing Capacity Market

Procurement optimization function

PUBLIC DOCUMENTATION

Version: 03 / 01 / 2025

Table of Contents

List of abbreviations

aFRR	Automatic Frequency Containment Reserve
BBCM	Baltic Balancing Capacity Market
BSP	Balancing Service Provider
CZC	Cross-Zonal Capacity
DRR	Demand Reduction Resource
FCR	Frequency Containment Reserve
FRR	Frequency Restoration Reserve
LFC	Load-Frequency Control
MCP	Market Clearing Price
MTU	Market Time Unit
mFRR	Manual Frequency Containment Reserve
NTC	Net Transfer Capacity
TSO	Transmission System Operator

List of notations

Sets

$\mathcal{B}$	Set of directional borders. Each border $b \in \mathcal{B}$ is a pair of areas $(m, n) \in \mathcal{N} \times \mathcal{N}$. For every physical border, two directional borders are created in $\mathcal{B}$. For example, the physical border between area m and area n requires adding entries (m, n) and (n, m) to $\mathcal{B}$.
$\mathcal{G}$	Set of exclusive groups. Each exclusive group $g \in \mathcal{G}$ contains a set of mutually exclusive orders $o_1, o_2, o_3, \ldots \in \mathcal{O}$.
$\mathcal{L}$	Set of inclusive links. Each inclusive link contains a pair of mutually inclusive orders $o_1, o_2 \in \mathcal{O}$.
$\mathcal{N}$	Set of areas. In practice, $\mathcal{N} = \{\text{Estonia}, \text{Latvia}, \text{Lithuania}, \text{Finland}, \text{Sweden}, \text{Poland}\}$.
$\bar{\mathcal{N}}$	Set of areas with the addition of the marketplace for the global Baltic requirement. In practice, $\bar{\mathcal{N}} = \{\text{Estonia}, \text{Latvia}, \text{Lithuania}, \text{Finland}, \text{Sweden}, \text{Poland}, \text{Baltic}\}$.
$\mathcal{O}$	Set of orders.
$\mathcal{O}_n$	Set of orders belonging to area $n \in \mathcal{N}$.
$\mathcal{O}_{np}$	Set of orders of product $p \in \mathcal{P}$ belonging to area $n \in \mathcal{N}$.
$\mathcal{O}_p$	Set of orders of product $p \in \mathcal{P}$.
$\mathcal{O}^{\mathrm{BR}}$	Set of sell orders originating from Backup Resources.
$\mathcal{O}^{\mathrm{DRR}}$	Set of sell orders originating from Demand Reduction Resources.
$\mathcal{O}^S$	Set of sell orders.
$\mathcal{O}^B$	Set of buy orders.
$\mathcal{O}_{np}^S$	Set of sell orders of product $p \in \mathcal{P}$ belonging to area $n \in \mathcal{N}$.
$\mathcal{O}_p^S$	Set of sell orders of product $p \in \mathcal{P}$.
$\mathcal{O}^B$	Set of buy orders.
$\mathcal{O}_{\mathrm{Baltic},p}^B$	Set of buy orders of product $p \in \mathcal{P}$ belonging to the Baltic perimeter, that is the buy orders expressing the global Baltic demand for product p.
$\mathcal{P}$	Set of products. In practice, the set of products is $\mathcal{P} = \mathcal{P}^{\mathrm{R}} \cup \{\text{ENERGY}\}$.
$\mathcal{P}^{\mathrm{R}}$	Set of reserve products. A product is the combination of a reserve type (FCR, aFRR, mFRR – also referred to as the process type) and a reserve direction (symmetrical, up, down). The exhaustive list of products is: FCR (symmetrical), aFRR up, aFRR down, mFRR up, mFRR down. That is $\mathcal{P} = \{\text{FCR}, \text{aFRRup}, \text{aFRRdown}, \text{mFRRup}, \text{mFRRdown}\}$.
$\mathcal{P}^{\mathrm{FRR}}$	Set of FRR products. That is $\mathcal{P}^{\mathrm{FRR}} = \{\text{aFRRup}, \text{aFRRdown}, \text{mFRRup}, \text{mFRRdown}\}$.
$\mathcal{T}$	Set of Market Time Units.
$\mathcal{T}_{\mathrm{o}}$	Set of Market Time Units covered by order $o \in \mathcal{O}$.

Constants

α_{nt}	€/(MWh)²	Rate of change of the energy price with change in the area net position for area $n \in \mathcal{N}$ on MTU $t \in \mathcal{T}$. If the area exports 1 additional MW, the price

		increases by α_{nt}. If the area imports one additional MW, the price decreases by α_{nt}.
β	-	Percentage of first level CZC available for allocation to reserve sharing. Default value at the time of writing is 50%.
M_o	-	Number of MTUs in the maximum duration constraint of order $o \in \mathcal{O}$.
n_o	-	LFC area $n_o \in \mathcal{N}$ where order o offers reserve volume.
$\mathrm{NTC}_{m\to n,t}$	MW	Net Transfer Capacity from area $m \in \mathcal{N}$ to area $n \in \mathcal{N}$ on period $t \in \mathcal{T}$.
Π_p^L	€/MW/h	Lower bound on the MCP for product $p \in \mathcal{P}$. Mind that the units should be €/MWh for the Energy product.
Π_p^U	€/MW/h	Upper bound on the MCP for product $p \in \mathcal{P}$. Mind that the units should be €/MWh for the Energy product.
P_{nt}^{ENERGY}	€/MWh	Initial energy price in area $n \in \mathcal{N}$ on MTU $t \in \mathcal{T}$.
p_o	-	Product $p_o \in \mathcal{P}^{\mathrm{R}}$ for which order o offers reserve volume.
P_{ot}	€/MW/h	Price of order $o \in \mathcal{O}$ on period $t \in \mathcal{T}_o$.
Q_{ot}^U	MW	Maximum quantity offered by of order $o \in \mathcal{O}$ on period $t \in \mathcal{T}_o$.
Q_{ot}^L	MW	Minimum quantity offered by of order $o \in \mathcal{O}$ on period $t \in \mathcal{T}_o$.
R_o	-	Number of MTUs in the resting duration constraint of order $o \in \mathcal{O}$.
$T_{m\to n,t}$	€/MWh	Markup to be added to the forecasted value of CZC on border $b = (m,n) \in \mathcal{B}$ and MTU $t \in \mathcal{T}$. Details about how the values of the markups are determined can be found in Article 6 of the "Methodology for the market-based allocation process of cross-zonal capacity for the exchange of balancing capacity for the Baltic CCR".
V_{nt}	MW	Initial energy net position of area $n \in \mathcal{N}$ on MTU $t \in \mathcal{T}$.

Variables

c_{np}	MW	Curtailment of the reserve requirement in product $p \in \mathcal{P}^{\mathrm{R}}$ and perimeter $n \in \bar{\mathcal{N}}$.
$\Delta_{m\to n,t,\mathrm{up}}$	MW	Increase of the CZC available for upward reserve products beyond the first level of CZC from area $m \in \mathcal{N}$ to area $n \in \mathcal{N}$ on period $t \in \mathcal{T}$.
$\Delta_{m\to n,t,\mathrm{down}}$	MW	Increase of the CZC available for downward reserve products beyond the first level of CZC from area $m \in \mathcal{N}$ to area $n \in \mathcal{N}$ on period $t \in \mathcal{T}$.
$f_{m\to n,p,t}$	MW	Flow of product $p \in \mathcal{P}$ on border $b = (m,n) \in \mathcal{B}$ and MTU $t \in \mathcal{T}$. For every pair of areas m and n with a common border, $(m,n) \in \mathcal{B}$ and $(n,m) \in \mathcal{B}$. Thus, two flow variables are used per product and MTU on the border: $f_{p,m\to n,t}$ and $f_{p,n\to m,t}$. Flow variables are non-negative: $f_{p,m\to n,t} \geq 0$.
π_{npt}	€/MW/h	Market clearing price for product $p \in \mathcal{P}$ in area $n \in \mathcal{N}$ on period $t \in \mathcal{T}$. Mind that the units of the price should be €/MWh for the Energy product.
q_{ot}	MW	Quantity accepted in order $o \in \mathcal{O}$ over MTU $t \in \mathcal{T}$. Positive for sell orders, negative for buy orders.

$s_{n,p,p',t}$	MW	Volume of product $p \in \mathcal{P}^{\mathrm{FRR}}$ used to substitute to product $p' \in \mathcal{P}^{\mathrm{FRR}}$ over MTU $t \in \mathcal{T}$ in area $n \in \mathcal{N}$. When non-zero, the substitute product p is matched against the requirement in the substitutable product p'. Typically, p could be aFRR up while p' could be mFRR up. However, the backward substitution of the more capable product by the less capable product is also possible in case of scarcity of supply in the more capable product. Transfer of volumes via substitutability implies that $s_{n,p,p',t} = -s_{n,p',p,t}$.
u_{ot}	-	Binary selection variable for order $o \in \mathcal{O}$ over MTU $t \in \mathcal{T}_o$. 1 if order o is accepted over MTU $t \in \mathcal{T}_o$ and 0 otherwise.
v_{nt}	MW	Volume accepted in the energy product in area $n \in \mathcal{N}$ on MTU $t \in \mathcal{T}$.

1 Introduction

This document provides a description of the N-SIDE Power Matching Algorithm, used to clear the Baltic Balancing Capacity Market (BBCM) auctions for the procurement of Frequency Containment Reserve (FCR) capacity and Frequency Restoration Reserve (FRR) capacity. The purpose of this supporting document is to provide clarity and transparency on technical aspects of the procurement optimization function (or clearing algorithm), including the objective function of the algorithm, co-optimisation, the order format and associated functionalities, and the approach to determining clearing prices.

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2 Overview

The BBCM is used by Baltic Transmission System Operators (TSOs) for the coupled day-head procurement of FCR and FRR balancing capacity. While two separate auctions are held for the procurement of FCR capacity and FRR capacity, each of them two optimizes the simultaneous procurement of the reserve capacity across all three Baltic countries and leverages their mutual connections.

FCR auction. The FCR auction organizes the procurement of reserve capacity for a single symmetrical (*i.e.*, upward and downward) product and allows for transfers of quantities between LFC areas without requiring Cross-Zonal Capacity (CZC) to be allocated.

FRR auction. The FRR auction organizes the procurement of reserve capacity for 4 unidirectional products.

- Automatic Frequency Reserve Restoration up (aFRR up).
- Automatic Frequency Reserve Restoration down (aFRR down).
- Manual Frequency Reserve Restoration up (mFRR up).
- Manual Frequency Reserve Restoration down (mFRR down).

The FRR auction allows for sharing of the procured reserve capacity across LFC areas and requires CZC to be allocated for that purpose. However, the auction should allocate CZC for the sharing of reserve only if it generates a higher value than the expected valuation of this CZC on the energy market. Therefore, an approximation of the energy market is also included in the auction via the introduction of a fifth product, the Energy product. The modelling of the energy market requires to also include the neighboring Finnish, Swedish (SE4 only) and Polish bidding zones in the auction set of areas (these areas will provide non-empty order books for the Energy product only).

MTU. Each auction carries out the procurement of reserve capacity for the entire next delivery day. The delivery day is divided into a set of time periods of equal length referred to as Market Time Units (MTUs).

Local and global capacity requirements. In every LFC area, the responsible TSO will specify its need in each of the reserve capacity products for each MTU. There will thus typically be one reserve capacity requirement per Baltic LFC area, per reserve product, per MTU. Additionally, a global requirement for the capacity procured in each reserve product might apply across the entire Baltic LFC block. The global requirement sets a lower bound on the total capacity to be procured in the Baltic region (regardless of its location) in the specified product.

Welfare maximization. The primary objective of the reserve auctions is to maximize the social welfare generated by the selection of BSP sell orders. The social welfare is equal to the sum of the surpluses generated by all agents participating in the auction.

- *Surplus of the BSPs orders*. The surplus of an order is the difference between the revenue that it generates by receiving the auction clearing price for its selected volume(s) and the cost that is incurred to them by the selected volume(s) as per the bidding price(s) of the order.
- *Surplus of the buyers*. The surplus of the buyers is the difference between the valuation that the buyers have for the procured volume and the procurement cost.
- *Congestion revenues*. The congestion revenues are the revenues generated by shipping of volumes across the network between areas with different prices. In the context of auctions for reserve capacity with multiple oligopolistic buyers (TSOs), the surplus of the buyer and the congestion revenues can merely be considered as two different parts originating from one same cake of which apportionment depends on the settlement rules between TSOs.

It can be shown that maximizing the sum of the individual surpluses of all agents amounts to maximizing the social welfare and reduces to maximizing the difference between the valuation that the buyers have for the procured volumes and the cost that the selected volumes incur to the BSPs according to their order's price(s).

3 Sell orders

This section introduces the structure of sell orders and explains how their properties are accounted for in the auction rules. It also clarifies what guarantees are offered on the quantities selected in the sell orders and how they relate to the market clearing prices.

3.1 Structure of sell orders

Sell orders are essentially made of a time series of price-quantity pairs. Table 1 describes the structure of sell orders input data and some of the validation rules applied to sell orders.

Table 1 - Structure of sell orders input data for sell order o.

Property	Notation	Units	Description
Product	$p_o \in \mathcal{P}^{\mathrm{R}}$	-	Product for which order o offers reserve capacity. The exhaustive list of supported products is: FCR symmetrical (up and down), aFRR up, aFRR down, mFRR up, mFRR down.
LFC area	$n_o \in \mathcal{N}$	-	LFC area where the order offers reserve capacity (Estonia, Latvia, or Lithuania – also referred to as the connecting domain).
Divisible	-	-	True if the order is divisible and false otherwise. For each MTU over which it offers a quantity, a divisible order can be accepted for any volume between the corresponding minimum quantity (included), or 0 (included) if no minimum quantity is specified, and the corresponding maximum quantity (included). For each MTU over which it offers a quantity, an indivisible order must either be fully rejected or accepted for exactly the specified quantity.
Block	-	-	True if the order is a block order and false otherwise. A block order must be accepted for the same quantity on all MTUs over which it specifies a non-zero quantity. Therefore, it is required that quantities specified by a block order are identical over all MTUs covered by the order. MTUs covered by a block order are also required to be consecutive. The Block flag is thus ineffective for orders covering a single MTU. An order that is not block can have its offered quantities accepted independently for the different MTUs it covers.
Linked bids identification	$l_o \in \mathcal{L}$	-	An order can be linked with another order covering the exact same set of MTUs. For every MTU, both orders must offer the same quantity (but the offered quantity can change across MTUs if the orders are not block). Only two orders can have the same link identifier l_o (*i.e.*, one order can only be linked with one other order). Linking can be used to link aFRR or mFRR orders of opposite directions, while aFRR and mFRR orders are not allowed to be linked to each other. For every MTU, linked orders will be accepted for the same quantity (but the quantity can change across MTUs if the orders are not block).

Property	Notation	Units	Description
Exclusive bids identification	$g_o \in \mathcal{G}$	-	An order can specify an exclusive group. For each MTU, a maximum of one order from the exclusive group can be accepted. Exclusive linking between aFRR and mFRR is not allowed. An exclusive group must thus contain only aFRR orders, or only mFRR orders.
Resting constraint duration	R_o	-	Number of consecutive MTUs over which the order must be rejected after the acceptance status of the order changes from (fully or partially) accepted to rejected. For example, if an order with a resting constraint duration of 2 MTUs is accepted on MTU 5 and fully rejected on MTU 6, then no quantity can be accepted in the order on MTU 7. Some quantity can be accepted again starting from MTU 8 (included) as the resting duration of 2 MTUs has elapsed. Note that a resting duration of 1 MTU is thus not an effective constraint. Additionally, for the resting constraint duration to be effective, the order should specify minimum quantities strictly above 1 MW for each MTU that it covers. Indeed, if the minimum quantity is 0 (or unspecified), the order can be considered accepted on a MTU even if the quantity matched on that MTU is 0 MW. The resting constraint duration would then not have to be applied on the next MTU(s). For technical reasons linked to TSO systems, the same applies if the minimum quantity is 1 MW. Block orders cannot specify a resting constraint duration.
Maximum constraint duration	M_o	-	Maximum number of consecutive MTUs over which the order can be accepted. For example, if the maximum constraint duration is 2 MTUs, the order cannot be accepted for 3 consecutive MTUs. If the order is accepted on MTU 1 and MTU 2, the order cannot be accepted on MTU 3. Block orders cannot specify a maximum constraint duration.
Prices	$P_{ot}\ \forall\, t \in \mathcal{T}_o$	€/MW/h	For each MTU t in the set $\mathcal{T}_o$ of MTUs covered by order o, a price P_{ot} must be provided. For block bids, prices P_{ot} must be equal on all covered MTUs: $P_{ot}^U = P_{os}^U\ \forall\, s,t \in \mathcal{T}_o$.
Quantities	$Q_{ot}^U\ \forall\, t \in \mathcal{T}_o$	MW	For each MTU t in the set $\mathcal{T}_o$ of MTUs covered by order o, a maximum quantity Q_{ot}^U must be provided. That is the maximum quantity that can be accepted in order o over MTU t. For block bids, quantities Q_{ot}^U must be equal on all covered MTUs: $Q_{ot}^U = Q_{os}^U\ \forall\, s,t \in \mathcal{T}_o$.
Minimum quantities	$Q_{ot}^L\ \forall\, t \in \mathcal{T}_o$	MW	For each MTU t in the set $\mathcal{T}_o$ of MTUs covered by order o, a minimum quantity Q_{ot}^L can be provided. The order cannot be accepted for a quantity smaller than the minimum quantity, but it can be fully rejected (0 MW accepted).

Property	Notation	Units	Description
			For block orders, minimum quantities Q^L_{ot} must be equal on all covered MTUs: $Q^L_{ot} = Q^L_{os}\ \forall\ s,t \in \mathcal{T}_o$. If a Divisible order does not provide a minimum quantity, the default value is 0 MW. A non-Divisible order will automatically use its quantities (Q^U_{ot}) as minimum quantities.

3.2 Selection of sell orders

This section describes the selection criteria and constraints for sell order $o \in \mathcal{O}$.

Variables. Sell orders use the following variables.

- q_{ot}: quantity (MW) accepted in order o over MTU $t \in \mathcal{T}_o$. For sell orders, q_{ot} is non-negative.
- u_{ot}: binary selection variable. 1 if order o is accepted over MTU $t \in \mathcal{T}_o$ and 0 otherwise.

Contribution to welfare. The selection of sell orders is primarily guided by the welfare maximization (see Section 2). Using the variables introduced above, the contribution of sell order o to the welfare is

$$-\sum_{t\in\mathcal{T}_o} P_{ot} q_{ot}.$$

Constraints. Several constraints must be satisfied by the realized selection of sell orders and market clearing prices.

First, the acceptance flag must be either 0 or 1 on every MTU.

$$u_{ot} \in \{0,1\} \qquad \forall\ t \in \mathcal{T}_o$$

Second, the minimum and maximum quantities must not be violated on any MTU over which the order is accepted.

$$Q^L_{ot} u_{ot} \le q_{ot} \le Q^U_{ot} u_{ot} \qquad \forall\ t \in \mathcal{T}_o$$

Third, block orders must be accepted for the same quantity on all MTUs.

$$q_{ot} = q_{os} \quad \text{(if order } o \text{ is a block order)} \qquad \forall\ s,t \in \mathcal{T}_o$$

Fourth, the quantity accepted in the order linked to order o must be equal to the quantity accepted in order o, for every MTU. If order o is linked to order o' (*i.e.*, $l_o = l_{o'}$), then the following constraints are applied.

$$q_{ot} = q_{o't} \qquad \forall\ t \in \mathcal{T}_o \cap \mathcal{T}_{o'}$$

Fifth, only one order can be accepted per MTU among all orders belonging to the same exclusive group. If order o belongs to exclusive group g_o, order o is thus involved in the following constraint.

$$\sum_{o'|g_{o'}=g_o} u_{o',t} \le 1 \qquad \forall\ t \in \cap_{o'|g_{o'}=g_o} \mathcal{T}_{o'}$$

Sixth, the maximum duration constraints of order o are applied as follows.

$$\sum_{s\in\{t,\dots,t+M_o\}\cap\mathcal{T}_o} u_{os} \le M_o \qquad \forall\ t \in \mathcal{T}_o$$

Seventh, the resting duration constraints of order o are applied as below. It must be highlighted that there is a slight abuse of notations as $u_{o,t+1}$ might not exist if $t+1 \notin \mathcal{T}_o$, in which case $u_{o,t+1}$ should be replaced by 0.

$$u_{os} \le 1 - (u_{ot} - u_{o,t+1}) \qquad \forall\ t \in \mathcal{T}_o, \forall s \in \{t+2,\dots,t+R_o\} \cap \mathcal{T}_o$$

Finally, the surplus of sell order o should be non-negative on every MTU where it is accepted, unless the negative surplus can be justified by the combination of the transfers of surplus received from (1) the linked order, (2) other MTUs if o is a block order, and (3) other MTUs via the resting duration constraints. In other words, sell orders should not be paradoxically accepted after transfers of surpluses are accounted for.

For (fully or partially) accepted non-block orders, the surplus is guaranteed to be non-negative on each MTU over which the order is (fully or partially) accepted. For every $t \in \mathcal{T}_o$ such that $u_{ot} = 1$,

$$q_{ot}\left(\pi_{n_o,p_o,t} - P_{ot}\right) + \tau^L_{l_o,o,t} + \underbrace{\sum_{s=t+1}^{t+R_o-1} \tau^R_{o,t-1,s} u_{o,s}}_{(a)} - \underbrace{\sum_{s=t+2}^{t+R_o} \tau^R_{o,t,s} u_{o,s}}_{(b)} - \underbrace{\sum_{s=t-R_o}^{t-2} \tau^R_{o,s,t}}_{(c)} \geq 0. \qquad \forall t \in \mathcal{T}_o \; | u_{ot} = 1$$

In this equation,

- $\tau^L_{l_o,o,t}$ is the transfer of money received via link l_o from the linked order if positive and sent to the linked order if negative.
- $\tau^R_{o,t,s}$ is the transfer of money received from (if positive) or sent to (if negative) other MTUs via the resting duration constraint for indexes t and s. Specifically, amount $\tau^R_{o,t,s}$ is positively contributing on MTU $t+1$ and negatively contributing on MTU t and MTU s. The three terms can be interpreted as follows.
    - (a) Transfers received from the $R_o - 1$ following MTUs so that the order is accepted on this MTU.
    - (b) Transfers given to the (up to $R_o - 1$) future MTUs to finance acceptance of the order on that MTU and avoid shutdown from this MTU (t).
    - (c) Transfers given to the $R_o - 1$ previous MTUs to finance acceptance of the order on that MTU and avoid shutdown until this MTU.

It should be highlighted that the last equation might occasionally be very slightly violated due to technicalities related to the rounding of prices and volumes. While slight paradoxical acceptance is thus possible at the level of individual MTUs, the order should still cumulate a non-negative revenue over the entire day.

For (fully or partially) accepted block orders, the cumulated surplus across all MTUs is guaranteed to be non-negative:

$$\sum_{t \in \mathcal{T}_o} \left(q_{ot}\left(\pi_{n_o,p_o,t} - P_{ot}\right) + \tau^L_{l_o,o,t} \right) \geq 0.$$

3.3 Paradoxes

While paradoxical acceptance is not allowed, paradoxical rejection is. A sell order can be rejected while its virtual surplus (*i.e.*, the surplus it would make if it was accepted and remunerated as per the published market clearing prices) is positive. It can be proven that avoiding both paradoxical acceptance and paradoxical rejection is not possible in the presence of indivisible orders.

The following set of orders will produce a market price of 15 €/MW/h and paradoxical rejection of the last order. Paradoxical rejection is due to the indivisibility of the accepted order.

- Demand requirement of 10 MW (accepted)
- Indivisible sell order of 10 MW at 15€/MW/h (accepted)
- Sell order of 9 MW at 10€/MW/h (rejected)

The following set of orders will produce a market price of 15 €/MW/h and paradoxical rejection of the last order. Paradoxical rejection is due to the indivisibility of the rejected order.

- Demand requirement of 10 MW (accepted)
- Sell order of 10 MW at 15€/MW/h (accepted)
- Indivisible sell order of 16 MW at 10€/MW/h (rejected)

3.4 Demand Reduction Resources

TSOs can submit sell orders originating from Demand Reduction Resources (DRR) to artificially reduce the TSO demand. These orders are simple orders: they comply with the sell order structure introduced previously but implement additional restrictions.

- DRR orders are always divisible.
- DRR orders are not block.
- Price of DRR orders is always 0 €/MW/h on all MTUs.
- The minimum quantity of DRR orders is always 0 MW or unspecified on all MTUs.

Because of their price of 0 €/MW/h, DRR orders are generally accepted before any order with a non-zero price is accepted. DRR orders are also treated differently during the computation of the market clearing prices as they don't contribute to the procurement cost (see Section 9).

3.5 Backup Resources

Orders from Backup Resources can be accepted only if the orders from Primary Resources (that is standard BSP orders) and Demand Reduction Resources cannot cover the TSOs demand. The total volume accepted in Backup Resources is minimized as explained in Section 8.

Backup Resources orders are simple orders, complying with the previously introduced sell order structure, but implementing some additional restrictions.

- Backup Resources orders are always divisible.
- Backup Resources orders are not block.
- Price of Backup Resources orders is always 0 €/MW/h on all MTUs.
- The minimum quantity of Backup Resources orders is always 0 MW or unspecified on all MTUs.

Because of their price of 0 €/MW/h, Backup Resources don't set the market clearing price, in the sense that they are not the marginal order unless the market clearing price is 0 €/MW/h. Backup Resources orders are also treated differently during the computation of the market clearing prices as they don't contribute to the procurement cost (see Section 9).

4 Buy orders

This section introduces the structure of buy orders and explains how their properties are accounted for in the auction rules. It also clarifies what guarantees are offered on the quantities selected in the buy orders.

4.1 Structure of buy orders

Buy orders have the same structure as sell orders (see Section 3.1) except that they implement the following restrictions.

1. Buy orders are divisible. It is always possible to procure less reserve than the TSO requirement.
2. Buy orders cannot be block orders.
3. Buy orders cannot be linked (l_o).
4. Buy orders cannot belong to an exclusive group (g_o).
5. Buy orders cannot specify any maximum duration constraint (M_o).
6. Buy orders cannot specify any resting duration constraint (R_o).
7. Buy orders are inelastic and therefore don't specify a price (P_{ot}).
8. Buy orders cannot specify any minimum quantity (Q^L_{ot}).
9. Buy orders cannot specify a non-zero quantity (Q^U_{ot}) for more than one MTU ($|\mathcal{T}_o| = 1$). One buy order per MTU is thus needed to procure reserve over several MTUs.

A buy order thus reduces to a product p_o, an area n_o, and a quantity Q^U_{ot}.

The area (connecting domain) of buy orders also supports the Baltic perimeter (on top of the values supported by sell orders, that is the 3 Baltic LFC areas in $\mathcal{N}$). A buy order defined with Baltic perimeter can be matched against sell volumes procured in any of the Baltic areas. For each MTU and reserve product, there will typically be one buy order per LFC area and one buy order for the Baltic LFC block.

4.2 Selection of buy orders

Buy orders are inelastic and should always be selected in full, except in case of lack of counterparty sell volume. See Section 8 for more details on the minimization of the curtailment in the requirement for reserve capacity.

Variables. Buy orders use the following variables.

- q_{ot}: quantity (MW) accepted in order o over MTU $t \in \mathcal{T}_o$. For buy orders, q_{ot} is non-positive (convention).

Contribution to welfare. Buy orders don't contribute to the welfare maximization as they are inelastic. That is equivalent to assuming that their contribution to the objective is $\sum_{t \in \mathcal{T}_o} M|q_{ot}|$ with M an arbitrarily large constant.

Constraints. The maximum quantity must not be violated on any MTU over which the order is accepted.

$$0 \leq |q_{ot}| \leq Q^U_{ot} \qquad \forall\, t \in \mathcal{T}_o$$

The order can be accepted at any quantity in between these bounds. In practice, non-full acceptance means there is curtailment of the reserve capacity requirement.

5 Approximation of the energy market CZC valuation

Because the FRR auction allocates CZC to the sharing of reserve (see Section 2), it must take into consideration the valuation of the CZC by the energy market and only allocate CZC to the extent that the valuation of the CZC for reserve capacity sharing is higher than the valuation of the CZC by the energy market. Therefore, the FRR auction must evaluate the value of the CZC on the energy market. This section describes the model used to approximate the energy market.

The model for the energy markets covers not only the Baltic bidding zones but also the neighboring bidding zones, namely Finland, Sweden (SE4 only) and Poland. Input data and output data are thus provided for these countries as well. In particular, it means that the import-export from-to these countries is captured within the model and influences the value of the CZC between Baltic countries.

Constants. The following constants are inputs of the model.

- α_{nt}: Sensitivity of the energy price to changes in the area net position for area $n \in \mathcal{N}$ on MTU $t \in \mathcal{T}$. If the area exports 1 additional MW, the price increases by α_{nt}. If the area imports one additional MW, the price decreases by α_{nt}.
- P_{nt}^{ENERGY}: The initial energy price in area $n \in \mathcal{N}$ on MTU $t \in \mathcal{T}$. This price can be understood as an initial guess as per where the energy market will settle.
- $T_{m\to n,t}$: Markup to be added to the forecasted value of CZC on border $b = (m, n) \in \mathcal{B}$ and MTU $t \in \mathcal{T}$. Details about how the values of the markups are determined can be found in Article 6 of the "Methodology for the market-based allocation process of cross-zonal capacity for the exchange of balancing capacity for the Baltic CCR".
- V_{nt}: The initial energy net position of area $n \in \mathcal{N}$ on MTU $t \in \mathcal{T}$. If positive (resp. negative), the area is expected to export towards (resp. import from) other areas. This position can be understood as an initial guess as per where the energy market will settle.

Variables.

- v_{nt}: Energy net position of area $n \in \mathcal{N}$ on MTU $t \in \mathcal{T}$. This position can be seen as a correction of V_{nt}.
- $f_{m\to n,\text{ENERGY},t}$: Flow of Energy on border $b = (m, n) \in \mathcal{B}$ and MTU $t \in \mathcal{T}$.

Contribution to welfare. The valuation of the CZC by the energy market is modelled by adding the following terms to the welfare maximization objective function. The left-hand side terms corresponds to the surplus of the linear curve defined in the volume-price domain by the point $(V_{nt}, P_{nt}^{\text{ENERGY}})$ and the coefficient α_{nt}. The right-hand side terms are the markups (tariffs on lines).

$$\sum_{n\in\mathcal{N}} \sum_{t\in\mathcal{T}} \left(P_{nt}^{\text{ENERGY}} (v_{nt} - V_{nt}) + \frac{1}{2} \alpha_{nt} (v_{nt} - V_{nt})^2 \right) - \sum_{b\in\mathcal{B}|b=(m,n)} \sum_{t\in\mathcal{T}} T_{m\to n,t} f_{m\to n,\text{ENERGY},t}$$

Because the linear curve of marginal cost cannot be paradoxically accepted or rejected, the estimated energy price in area $n \in \mathcal{N}$ on MTU $t \in \mathcal{T}$ will be given by

$$\pi_{n,\text{ENERGY},t} = P_{nt}^{\text{ENERGY}} + \alpha_{nt}(v_{nt} - V_{nt}).$$

The valuation of the CZC from area m to area n by the energy market is given by the market price spread $\pi_{n,\text{ENERGY},t} - \pi_{m,\text{ENERGY},t}$. For the sharing of balancing capacity to take precedence in the capacity allocation, it must have an associated value higher than the energy market price spread minus the markup.

Finally, note that the balance of the energy product is discussed and enforced in Section 6.3.

6 Supply-demand balance equations and substitution

For each area n and product p (including the energy product), the auction model enforces that the accepted volumes on the demand side are adequately matched by the volumes accepted in the supply side by enforcing the supply-demand balance equations.

6.1 Types of balances

For the reserve capacity products, the model allows for procuring more than the total demand volume, if it is necessary to avoid unmatched demand or to increase the welfare. That is typically needed in presence of indivisible orders (accepting an indivisible order of which volume is larger than the demand requirement is a better outcome than accepting no volume at all). Procuring beyond the total demand volume is referred to as overholding or over-procurement. Overholding is modelled by using inequalities in the supply-demand balance equations of the reserve products introduced later in this section. The supply-demand balance equations for the Energy product on the other hand will use equalities as a perfect balance of Energy is needed.

It might also happen that the demand requirement cannot be matched in some or all areas. In such a case (see Section 8), the curtailed volume for the area and product c_{np} appears in the balance equation.

Transfer or sharing of volumes is allowed to take place across areas. For every pair of areas $b = (m, n) \in \mathcal{B}$ having a common border, a non-negative flow variable $f_{m\to n,p,t} \geq 0$ is introduced to model the volume of product $p \in \mathcal{P}$ flowing from area m to area n (mind that variable $f_{n\to m,p,t} \geq 0$ is thus also introduced in the model as it also holds that $(n, m) \in \mathcal{B}$). The following subsections explain how the flow variables relate to each other and to the quantities accepted in the orders.

6.2 Transfer between products via substitution

Volume of product $p \in \mathcal{P}^{\mathrm{FRR}}$ might be used to substitute for product $p' \in \mathcal{P}^{\mathrm{FRR}}$, in particular when it increases the overall welfare. Typically, p could be aFRR up while p' could be mFRR up. However, the backward substitution of the more capable product by the less capable product is also possible in case of scarcity of supply in the more capable product.

Substitution is modelled by variables $s_{n,p,p',t}$, which is the volume of product $p \in \mathcal{P}$ used to substitute to product $p' \in \mathcal{P}$ over MTU $t \in \mathcal{T}$. When non-zero, the substitute product p is transferred towards the substitutable product p'. Transfer of volume via substitutability implies that $s_{n,p,p',t} = -s_{n,p',p,t}$.

$$s_{n,p,p',t} = -s_{n,p',p,t} \qquad \forall\, n \in \mathcal{N}, \forall\, p \in \mathcal{P}, \forall\, p' \in \mathcal{P}, \forall\, t \in \mathcal{T}$$

Not all forward and backward substitutions are allowed. For some combinations of p and p', variable $s_{n,p,p',t}$ is thus forced to 0. The allowed substitutions are:

- Substitute mFRR up (p') with aFRR up (p) is always allowed.
- Substitute mFRR down (p') with aFRR down (p) is always allowed.
- Substitute aFRR up (p') with mFRR up (p) is allowed only if the demand requirement for aFRR up cannot be met with aFRR up sell orders (see Section 8 for more details on the course of actions when demand requirements cannot be met).
- Substitute aFRR down (p') with mFRR down (p) is allowed only if the demand requirement for aFRR down cannot be met with aFRR down sell orders (see Section 8 for more details on the course of actions when demand requirements cannot be met).

There is no other type of meaningful substitution.

6.3 Transfer between areas

The FCR and Energy products implement transfer of volume between areas. Thus, 1 MW supplied in area A can be shipped to the connected area B. If so, the MW cannot be used to satisfy the local reserve or energy demand in area A.

For each area n, the balance equations for FCR and for Energy are written as follows (both products are assumed not to be involved in substitutions of any kind).

$$\sum_{o\in\mathcal{O}_{n,\text{FCR}}} q_{ot} - \sum_{b\in\mathcal{B}|b=(n,m)} f_{n\to m,\text{FCR},t} + \sum_{b\in\mathcal{B}|b=(m,n)} f_{m\to n,\text{FCR},t} + c_{n,\text{FCR}} \geq 0 \qquad \forall\, n\in\mathcal{N}, \forall\, t\in\mathcal{T}$$

$$v_{nt} - \sum_{b\in\mathcal{B}|b=(n,m)} f_{n\to m,\text{ENERGY},t} + \sum_{b\in\mathcal{B}|b=(m,n)} f_{m\to n,\text{ENERGY},t} = 0 \qquad \forall\, n\in\mathcal{N}, \forall\, t\in\mathcal{T}$$

Note that FCR capacity implements overholding while Energy does not, hence the former uses inequality while the latter uses equality. The last term in the FCR balance equation ($c_{n,\text{FCR}}$) is the curtailment of the requirement (see Section 8 for information on the minimization of the curtailment).

Because the volume flowing between areas m and n is transferred and not shared, the following constraints on the flows must be added (no bidirectional flows).

$$f_{m\to n,p,t} f_{n\to m,p,t} = 0 \qquad \forall\, (m,n)\in\mathcal{B}, \forall\, p\in\{\text{FCR}, \text{ENERGY}\}, \forall\, t\in\mathcal{T}$$

6.4 Sharing between areas

The aFRR up, aFRR down, mFRR up, and mFRR down products implement sharing of reserve between areas. Thus, 1 MW supplied in area A can be shipped to the connected area B to satisfy the reserve requirement in area B. The same MW can also be used to satisfy the reserve requirement in area A.

For each area n and FRR product $p\in\mathcal{P}^{\text{FRR}}$, the balance equation is written as follows.

$$\sum_{o\in\mathcal{O}_{np}} q_{ot} + \sum_{b\in\mathcal{B}|b=(m,n)} f_{m\to n,p,t} + \sum_{p\prime\in\mathcal{P}^{\text{FRR}}\setminus\{p\}} s_{np\prime pt} + c_{np} \geq 0 \qquad \forall\, n\in\mathcal{N}, \forall\, p\in\mathcal{P}^{\text{FRR}}, \forall\, t\in\mathcal{T}$$

The first term is the balance of local supply and demand. The second term is the volume shared by other areas. The third term is the net substitution volume (positive if volume is received from substitutable products, negative if this product is used as a substitute for other products). The last term is the curtailment of the requirement.

Because the procured volumes are shared between areas, it is not needed to prevent bidirectional flows (while it was required for products implementing transfers of volumes, see Section 6.3). In other words, area A can share with area B at the same time as area B shares with area A.

Additionally, the following constraints must be added for any product p implementing sharing of reserves to avoid the receiving area reshares a MW with the area from which this MW was received.

$$f_{m\to n,p,t} \leq \sum_{o\in\mathcal{O}^S_{mp}} q_{ot} + \sum_{k\in\mathcal{B}|k=(\bullet,m)\wedge k\neq(n,m)} f_{\bullet\to m,p,t} \qquad \forall\, (m,n)\in\mathcal{B}, \forall\, p\in\mathcal{P}^{\text{FRR}}, \forall\, t\in\mathcal{T}$$

This constraint bounds the volume shared from m to n by the local supply in m plus all the volume shared with m by all other areas except n. This constraint achieves the desired outcome under the assumption that the network topology has no cycles.

For the global Baltic LFC block, the balance equation requiring the global Baltic requirement for reserve capacity to be satisfied is written as follows.

$$\sum_{o\in\mathcal{O}^S_p} q_{ot} - \sum_{o\in\mathcal{O}^B_{\text{Baltic},p}} q_{ot} + c_{\text{Baltic},p} \geq 0 \qquad \forall\, p\in\mathcal{P}^{\text{FRR}}, \forall\, t\in\mathcal{T}$$

7 Capacity allocation

The FCR capacity auction does not implement limits on the transfers of FCR between areas because it is not required by nature of the FCR product.

On the other hand, the FRR products require Cross Zonal Capacity (CZC) to be allocated in case of sharing of reserve between two areas. Nonetheless, reserve products are in competition with the energy product for the acquisition of the transfer capacity between areas which is limited. The FRR auction provides an estimation for the valuation of the CZC by the energy product (see Section 5) to allocate the capacity to the product having the higher valuation for it. The following constraints are added to ensure that the total allocated capacities (or the expected allocated capacities in the case of the energy product) remain smaller than the Net Transfer Capacity (NTC).

$$f_{m\to n,\mathrm{aFRRup},t} + f_{m\to n,\mathrm{mFRRup},t} + f_{m\to n,\mathrm{ENERGY},t} \le \mathrm{NTC}_{m\to n,t} \qquad \forall\,(m,n) \in \mathcal{B},$$

$$f_{n\to m,\mathrm{aFRRdown},t} + f_{n\to m,\mathrm{mFRRdown},t} + f_{m\to n,\mathrm{ENERGY},t} \le \mathrm{NTC}_{m\to n,t} \qquad \forall\, t \in \mathcal{T}$$

Mind that the NTC is imposed using two constraints instead of a single and more constraining one of which left-hand side would gather all 5 terms of the left-hand sides of the two equations above. That is because imbalance netting takes place at activation stage. Indeed, if area n needs 1 MW of upward activation and are m needs 1 MW of downward activation, only 1 MW must be shipped from area m to area n, not 2 MW.

The total capacity allocated to FRR products is also not allowed to go beyond a certain percentage (β) of the NTC. At the time of writing this documentation, the default value for β is 50%. The limit on the capacity allocated to FRR products is imposed as follows.

$$f_{m\to n,\mathrm{aFRRup},t} + f_{m\to n,\mathrm{mFRRup},t} \le \beta\mathrm{NTC}_{m\to n,t} + \Delta_{m\to n,t,\mathrm{up}} \qquad \forall\,(m,n) \in \mathcal{B},$$

$$f_{n\to m,\mathrm{aFRRdown},t} + f_{n\to m,\mathrm{mFRRdown},t} \le \beta\mathrm{NTC}_{m\to n,t} + \Delta_{m\to n,t,\mathrm{down}} \qquad \forall\, t \in \mathcal{T}$$

Variables $\Delta_{m\to n,t,\mathrm{up}}$ and $\Delta_{m\to n,t,\mathrm{down}}$ are modelling possible relaxations in these constraints. The clearing algorithm will always attempt to set these variables to 0. However, in case of supply scarcity in some areas, meeting the requirement for reserve capacity might require consuming more network capacity than $\beta\mathrm{NTC}_{m\to n,t}$. Variables $\Delta_{m\to n,t,\mathrm{up}}$ and $\Delta_{m\to n,t,\mathrm{down}}$ are thus gradually increased until the reserve requirements can be met. There is however a maximum value allowed for these increases: at the time of writing this documentation, the maximum value allowed for the capacity allocated to the sharing of reserve is 70% of the NTC, meaning that $\Delta_{m\to n,t,\mathrm{up}}$ and $\Delta_{m\to n,t,\mathrm{down}}$ must be smaller than or equal to $0.2 * \mathrm{NTC}_{m\to n,t}$. See Section 8 for more information on the relaxation of the constraints.

As explained in Section 5, the valuation of the CZC from area m to area n by the energy market is given by the market price spread $\pi_{n,\mathrm{ENERGY},t} - \pi_{m,\mathrm{ENERGY},t}$ (as in the Day-Ahead Market). For the sharing of balancing capacity to take precedence in the capacity allocation, it must have an associated value higher than the energy market price spread minus the markup. In other words, the cost of local reserve procurement minus the cost (if any) of foreign reserve procurement (*i.e.*, the cost of increasing local procurement to reduce foreign procurement) must be higher than the energy market price spread minus the markup.

It is important to note that price spreads on a border are not necessarily harmonized across FRR products and energy product. That is because the shadow prices of the capacity constraints specified above are not the only contributors to the price differences. For example, the enforcement of the lower bounds on the flows at the individual product level (*e.g.*, $0 \le f_{m\to n,\mathrm{aFRRup},t}$) can create price differences that are not common to all products.

8 Infeasible requested volume

It might happen that sell orders submitted by BSPs are not sufficient to cover the reserve requirements (be it the individual requirements of the areas or the global Baltic requirement) on one or several MTUs.

In such case, the auction must activate the following measures in sequential order: 2nd level CZC, backup resources, substitution of the more capable product by the less capable product (*i.e.*, substitution of aFRR by mFRR), and finally curtailment of the reserve requirement. The clearing algorithm implements the activation of these measures by walking through them in reversed order and minimizing their respective usage. The algorithm proceeds as follows.

1) Minimize the total volume of curtailed demand with
   a) Total volume of curtailed demand free.
   b) Total volume of more capable product substituted by less capable product free.
   c) Total volume activated in Backup Resources free.
   d) Total amount of used 2nd level CZC free.
2) Minimize the total volume of more capable product substituted by less capable product with
   a) Total volume of curtailed demand fixed at its optimal value of point 1).
   b) Total volume of more capable product substituted by less capable product free.
   c) Total volume activated in Backup Resources free.
   d) Total amount of used 2nd level CZC free.
3) Minimize the total volume activated in Backup Resources with
   a) Total volume of curtailed demand fixed at its optimal value of point 1).
   b) Total volume of more capable product substituted by less capable product fixed at its optimal value of point 2).
   c) Total volume activated in Backup Resources free.
   d) Total amount of used 2nd level CZC free.
4) Minimize the total amount of used 2nd level CZC with
   a) Total volume of curtailed demand fixed at its optimal value of point 1).
   b) Total volume of more capable product substituted by less capable product fixed at its optimal value of point 2).
   c) Total volume activated in Backup Resources fixed at its optimal value of point 3).
   d) Total amount of used 2nd level CZC free.
5) Maximize the welfare with
   a) Total volume of curtailed demand fixed at its optimal value of point 1).
   b) Total volume of more capable product substituted by less capable product fixed at its optimal value of point 2).
   c) Total volume activated in Backup Resources fixed at its optimal value of point 3).
   d) Total amount of used 2nd level CZC fixed at its optimal value of point 4).

At each step, the criteria optimized in the previous step is fixed at its optimal value. Mind that some tolerances might be used in the process, which could incur some slight deviations from the theoretical optimal value.

9 Market clearing prices

For each MTU, each area, and each product, a uniform market clearing price is determined. This means that all accepted buy and sell orders for a given MTU, area, and product are settled at the same market clearing price. The uniform market clearing prices must comply with the acceptance criteria for sell orders introduced in Section 3.2 (surplus of accepted volumes must be non-negative). However, these conditions do not suffice to uniquely determine the market clearing prices corresponding to a welfare maximizing solution as there might still remain some indeterminacy. In such cases, the remaining price indeterminacy is lifted by minimization of the procurement cost, followed by minimization of the sum of the squared market clearing prices (the two steps being needed as the first one does not suffice to fully lift the indeterminacy). The procurement cost is fixed to its minimal value (with some tolerance) during the minimization of the sum of squared prices.

The minimization of the procurement cost writes as follows. Mind that the energy product does not intervene in the expression, and that the Backup resources and Demand Reduction Resources don't have any associated procurement cost.

$$\min_{\pi_{npt}} \sum_{o \in \mathcal{O}^S \setminus \mathcal{O}^{\mathrm{BR}} \setminus \mathcal{O}^{\mathrm{DRR}}} \sum_{t \in \mathcal{T}_o} q_{ot} \pi_{n_o, p_o, t}.$$

The minimization of the sum of the squared prices writes as follows. Mind that the energy product does not intervene in the expression so as not to impact the prices in case an area does not specify an energy curve (convergence of price spreads across products could in such case create an interference of the meaningless energy price within the computation of the prices of the reserve products).

$$\min_{\pi_{npt}} \sum_{n \in \mathcal{N}} \sum_{p \in \mathcal{P}^{\mathrm{R}}} \sum_{t \in \mathcal{T}} \pi_{npt}^2.$$

The minimization of the sum of the squared prices has a tendency to harmonize the prices across MTUs, products, and areas, while maintaining the minimal procurement cost.

Finally, technical price bounds are also enforced on the market clearing prices: $\Pi_p^L \leq \pi_{npt} \leq \Pi_p^U$. Procurement cost minimization could indeed lead to prices that don't fit within any arbitrarily large price bounds if they are not explicitly enforced at the optimization stage. To avoid arbitrarily large prices, procurement cost minimization is thus constrained by the enforcement of the technical price bounds. At the time of writing this documentation, the technical price bounds are

- $\Pi_p^L = 0$ €/MW/h and $\Pi_p^U = 4000$ €/MW/h for the reserve products. (FCR, aFRR up, aFRR down, mFRR up, mFRR down). In practice, the minimum and maximum market price bounds are the same for all the reserve products.
- $\Pi_p^L = -500$ €/MW/h and $\Pi_p^U = 4000$ €/MW/h for the energy products.

However, these bounds might take other values in the future. For example, the minimum and maximum market prices for the energy product have to be reflect any change in the minimum and maximum market price bounds on the European single day-ahead market.

10 Indeterminacy on sell orders acceptances

The welfare maximization program might accept several welfare-equivalent solutions which differ in the volume accepted in the individual sell orders. In such case, the clearing algorithm implements some rules to decide which solution to favor: (1) harmonization of the acceptance ratios of equivalent-but-volume orders, followed by (2) tie-breaking based on timestamps. This section provides more details on these rules. Any indeterminacy remaining after these rules are applied is lifted by the solver using pseudo-randomness.

10.1 Harmonization of acceptance ratios

Block sell orders covering several MTUs are equivalent-but-volume if

1) They belong to the same area n_o.
2) They are for the same product p_o.
3) They have the same prices P_{ot} (mind that a block order is already required to specify the same price on all MTUs).
4) They cover the same set of MTUs $\mathcal{T}_o$.
5) They don't belong to an exclusive group.

Acceptance ratios of equivalent-but-volume block sell orders (remember a block order as a single acceptance ratio common to all MTUs) are harmonized in case they cannot all be fully accepted. In other words, their respective acceptances are pro-rated proportionally to their bid's volume.

Non-block sell orders or single MTU sell orders are equivalent-but-volume on MTU $t \in \mathcal{T}$ (for which they both specify a non-zero quantity) if

1) They belong to the same area n_o.
2) They are for the same product p_o.
3) They have the same prices P_{ot}.
4) They don't belong to an exclusive group.
5) They don't belong to a maximum duration constraint.
6) They don't belong to a resting duration constraint.

Acceptance ratios of such equivalent-but-volume sell orders (remember a non-block order has one acceptance ratio per MTU) are harmonized per MTU in case they cannot be fully accepted over that MTU. In other words, their respective acceptances are pro-rated proportionally to their bid's volume.

Note that the harmonization of the acceptance ratios is only done on a best effort basis, and there are cases for which harmonization will not be possible. In particular, orders involved in inclusive links might prevent the full harmonization of acceptance ratios across equivalent-but-volume sell orders.

10.2 Timestamps tie-breaking

Two sell orders are equivalent-but-timestamp if

1) They belong to the same area n_o.
2) They are for the same product p_o.
3) They are both Divisible or both Indivisible.
4) They cover the same set of MTUs $\mathcal{T}_o$.
5) They don't specify a linked order.
6) They don't belong to an exclusive group.
7) They don't belong to a maximum duration constraint.
8) They don't belong to a resting duration constraint.
9) They have the same prices P_{ot} on all MTUs.
10) They have the same quantities Q_{ot}^U on all MTUs.
11) They have the same minimum quantities Q_{ot}^L on all MTUs.

If two sell orders are equivalent-but-timestamp, the quantities accepted in the order with the least recent timestamp are always higher than the quantities accepted in the order with the most recent timestamp.

11 Rounding

Because the clearing algorithm works with real numbers (or more precisely their binary representation), volumes accepted in buy and sell orders as well as market clearing prices can take non-integer values after welfare maximization has been performed. The final volumes selected in the buy and sell orders must be integer while the final market clearing prices must be multiples of 0.01 €/MW/h. Therefore, the results of the welfare maximization are rounded. This section describes the rounding process.

11.1 Rounding of market clearing prices

Prices are rounded up to the closest multiple of 0.01 €/MW/h. For example:

- If the unrounded price is 10.330 €/MW/h, the rounded price is 10.33 €/MW/h.
- If the unrounded price is 10.331 €/MW/h, the rounded price is 10.34 €/MW/h.
- If the unrounded price is 10.335 €/MW/h, the rounded price is 10.34 €/MW/h.
- If the unrounded price is 10.339 €/MW/h, the rounded price is 10.34 €/MW/h.
- If the unrounded price is 10.340 €/MW/h, the rounded price is 10.34 €/MW/h.
- If the unrounded price is -10.330 €/MW/h, the rounded price is -10.33 €/MW/h.
- If the unrounded price is -10.331 €/MW/h, the rounded price is -10.33 €/MW/h.
- If the unrounded price is -10.335 €/MW/h, the rounded price is -10.33 €/MW/h.
- If the unrounded price is -10.339 €/MW/h, the rounded price is -10.33 €/MW/h.
- If the unrounded price is -10.340 €/MW/h, the rounded price is -10.34 €/MW/h.

Rounding upward ensures that no sell order becomes paradoxically accepted with respect to the rounded prices. In the example of Figure 4, the market clearing price will be 1000 £/MW/h for Product 1 and 4000/3 £/MW/h for Product 2.

11.2 Rounding of accepted volumes

For every sell order and MTU, the volume accepted in the sell order on the MTU is rounded up to the closest multiple of 1 MW.

- If the unrounded volume is 10 MW, the rounded volume is 10 MW
- If the unrounded volume is 10.1 MW, the rounded volume is 11 MW
- If the unrounded volume is 10.5 MW, the rounded volume is 11 MW
- If the unrounded volume is 10.9 MW, the rounded volume is 11 MW

11.3 Rounding of allocated capacities

First, allocated capacities are rounded to the closest. However, this might lead to the rounded accepted volumes and allocated capacities not to satisfy the reserve requirement by one or several rounding ticks. Therefore, the remaining available CZC is used to adjust the rounded allocated capacities in order to minimize the demand requirement that is unmatched due to rounding.